

Group 'A'

Attempt all the questions

[10×1=10]

Select the best alternative.

- If $A = \{1, 2, 3, 4, 5\}$ then the number of subsets of A are:
a) 32 b) 64 c) 16 d) 31
- The value of $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is equal to:
a) i b) -4 c) $-i$ d) 0
- A function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x + 1$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ defined by $g(x) = x^2$ then $f \circ g(x) =$
a) $2(x+1)$ b) $(x-1)^2$
c) $x^2 + 1$ d) $(x+1)^2$
- In an A.P., if $t_n = 2n+3$ then $S_{10} =$
a) 120 b) 130 c) 140 d) 110
- If x, y, z are in A.P., then e^{-x}, e^{-y}, e^{-z} are in
a) A.P. b) G.P.
c) H.P. d) Combination of A.P. and G.P.
- If any two rows of a determinant are identical then the value of the determinant is
a) 0 b) 1
c) -1 d) Can't determine
- If $A = \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ 5 & -3 \end{pmatrix}$, then the product of AB is
a) $\begin{pmatrix} -1 & 5 \\ 4 & -3 \end{pmatrix}$ b) $\begin{pmatrix} 13 & -5 \\ 1 & -2 \end{pmatrix}$
c) $\begin{pmatrix} -13 & 5 \\ -1 & 2 \end{pmatrix}$ d) $\begin{pmatrix} -2 & 6 \\ -5 & 0 \end{pmatrix}$
- If eccentricity of a conic $e = 1$, then the conic is
a) Ellipse b) Circle
c) Parabola d) Hyperbola
- The vector perpendicular to both \vec{a} and \vec{b} is
a) $\vec{a} + \vec{b}$ b) $\vec{a} \cdot \vec{b}$
c) $\vec{a} \times \vec{b}$ d) $\vec{a} - \vec{b}$
- How many number plates of vehicles consisting 4 different digits can be made out of integers 2, 3, 4, 5, 6 and 7?
a) 120 b) 360
c) 84 d) 15

Group 'B'

Attempt any six questions:

[6×5=30]

1. If $n(U) = 360$, $n(A) = 240$, $n(B) = 160$, find the maximum value of $n(A \cap B)$ and the minimum value of $n(A \cap B)$. When will the value of $n(A \cap B)$ be maximum and find its value.

2. Find sum upto infinity of the series given by

$$1 + 3x + 5x^2 + 7x^3 + \dots \text{ to } \infty$$

3. Prove that:
$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix} = a^2(a+3)$$

4. Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 3+2x & \text{for } -\frac{3}{2} \leq x < 0 \\ 3-2x & \text{for } 0 \leq x < \frac{3}{2} \\ -3-2x & \text{for } x \geq \frac{3}{2} \end{cases} .$$

Find: a) $f(-\frac{3}{2})$ b) $f(1)$ c) $\frac{f(h)-f(0)}{h}$ for $0 \leq h < \frac{3}{2}$

5. A committee of 5 is to be formed out of 6 Gents and 4 Ladies. In how many ways can this be performed when,

(i) Exactly two ladies are included

(ii) At least two ladies are included

6. Show that $9x^2 + 4y^2 - 18x - 16y - 11 = 0$ represent the equation of an ellipse. Find its center and eccentricity.

7. If $x \in \mathbb{R}$ and a be any positive real number then prove that $|x| < a \Rightarrow -a < x < a$. Rewrite by using the absolute value sign for $-7 < x < 1$.

Group 'C'

Attempt any two questions:

[10×2=20]

8. What is a singular matrix? When does a matrix have its inverse? If

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 1 \\ 0 & 3 & -1 \end{bmatrix}, \text{ find its inverse.}$$

9. Define scalar product of three vectors. What is the geometrical meaning of scalar triple product? Find the volume of a parallelepiped whose concurrent edges are represented by the vectors

$$\vec{a} = \hat{i} + 2\hat{j} + 3\vec{k}, \vec{b} = 3\hat{i} + 4\hat{j} + -5\vec{k} \text{ and } \vec{c} = \hat{i} - 2\hat{j} + 3\vec{k}$$

10. a. What do you mean by a function? Find the domain and range of the following function defined in the set of real numbers (i.e. $f: \mathbb{R} \rightarrow \mathbb{R}$)
 $y = x^2 - 6x + 6$
- b. In how many ways can the letters of the word "MATHEMATICS" be arranged so that all the vowels are always together?